

# A simple hidden variable experiment

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**Abstract.** An experiment is described which proves, using single photons only, that the standard hidden variables assumptions (commonly used to derive Bell inequalities) are inconsistent with quantum mechanics. The analysis is very simple and transparent. In particular, it demonstrates that a classical wave model for quantum mechanics is not ruled out by experiments demonstrating the violation of the traditional hidden variable assumptions.

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# 1 Introduction

It is well-known (see, e.g., ASPECT [1], CLAUSER & SHIMONY [7], TITTEL et al. [28]) that the foundations of quantum theory can be tested by means of optical experiments verifying the violation of Bell inequalities; cf. BELL [5], PITOWSKY [22], and the recent review of hidden variable theories by GENOVESE [9].

In contrast to Bell inequalities which need strongly entangled two-photon states to give a contradiction with the quantum predictions, the experiment suggested in the following works with only simple optical equipment, using arbitrary single-photon states or even strong laser light. Of course, there is also entanglement involved – not between two photons but between the polarization and the spatial degrees of freedom of a single photon.

Moreover, the new experiment provides much sharper predictions than traditional Bell inequalities. It is much more robust since it can be carried out with strong laser light. Its very simple analysis gives new insight into the reason for the failure of hidden variable assumptions. In particular, the present analysis demonstrates that a classical wave model for quantum mechanics is not ruled out by experiments demonstrating the violation of the traditional hidden variable assumptions, also used in the derivation of Bell inequalities.

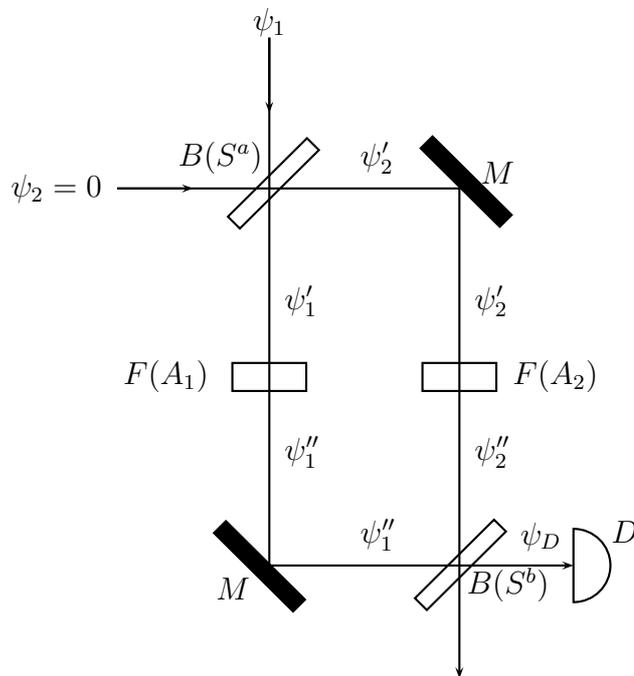
Other papers discussing experiments involving the entanglement of single photons include BABICHEV et al. [2], BARTLETT et al. [3], BEIGE et al. [4], CAN et al. [6], VAN ENK [8], GERRY [10], HARDY [11], IKRAM & SAIF [14], HESSMO et al. [12], KIM [16], LEE & KIM [17], PERES [21], SPREEUW [25], TAN et al. [27], WILDFEUER et al. [31]. In particular, HESSMO et al. [12] and WILDFEUER et al. [31] verify experimentally the prediction of single-particle nonlocality by TAN et al. [27], and BABICHEV et al. [2] discusses the detection loophole for single-particle Bell inequality violation. HILLMER & KWIAT [13] described a quantum eraser experiment which can be performed using ordinary laser light.

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## 2 The experiment

We consider a fixed, symmetric arrangement of optical instruments in the form of Figure 1. For describing the experiment in quantum mechanical terms, we use unnormalized wave functions  $\psi \in \mathbb{C}^2$  to denote the state of an ensemble of photons in a quasi-monochromatic beam of completely polarized light.  $\psi$  is scaled such that  $|\psi|^2 = \psi^* \psi$  equals the probability of presence of a photon in the beam in a fixed time interval  $[t_0, t_1]$ ; in particular, dark beams without any photons are described by  $\psi = 0$ . It is easily checked that optical filters

Figure 1: The hidden variable experiment



consisting of a combination of polarizers are then described by linear transformations of  $\psi$  of rank one, and that arbitrary such linear transformations  $\psi \rightarrow A\psi$  ( $A = uv^*$ ) can be realized as long as  $|u||v|$  is significantly smaller than 1; cf. JONES [15].

Each  $B(S)$  is a non-polarizing beam splitter with a fixed unitary scattering matrix  $S \in \mathbb{C}^{2 \times 2}$ , each  $M$  is a mirror, each  $F(A)$  is a linear filter transforming the unnormalized wave function  $\psi$  into  $A\psi$ , with an adjustable transformation matrix  $A$ , and  $D$  is a detector registering an incident photon with probability  $q$ . Note that by choosing the distances appropriately while keeping the symmetry of the paths, we can move the two filters (one under the control of Alice and the other under the control of Bob, in the traditional quantum communication terminology) as far apart as we like. Thus the experiment can be given a nonlocal touch, if desired.

The sources of the two beams are not drawn; they are assumed to produce completely polarized light described by the unnormalized wave functions  $\psi_k$  ( $k = 1, 2$ ). The wave functions are scaled such that initially

$$\psi_1^* \psi_1 + \psi_2^* \psi_2 = 1,$$

corresponding to the presence of just one photon in both beams together. The scattering

matrices of the two beam splitters are fixed in the experiment and given by

$$S^j = \begin{pmatrix} t_1^j & r_2^j \\ r_1^j & t_2^j \end{pmatrix} \quad (j = a, b),$$

where  $t_k^j$  are the transmission coefficients and  $r_k^j$  the reflection coefficients of the two beams; two input beams of the beam splitter with wave functions  $\psi_1$  and  $\psi_2$  are transformed into

$$\begin{pmatrix} \psi'_1 \\ \psi'_2 \end{pmatrix} = S^j \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} t_1^j \psi_1 + r_2^j \psi_2 \\ r_1^j \psi_1 + t_2^j \psi_2 \end{pmatrix};$$

cf. MANDEL & WOLF [19, Section 12.12].

We denote by  $p(A_1, A_2)$  the probability of detection of a photon in the given arrangement, where  $A_1$  and  $A_2$  are the transformation matrices for beam 1 and beam 2, respectively, and the second beam is initially dark. We analyze the expected dependence of the difference

$$\Delta(A_1, A_2) := p(A_1, A_2) - p(A_1, 0) - p(0, A_2). \quad (1)$$

on the arguments in two ways, first assuming a classical hidden variable model, and then assuming quantum mechanics. By recording enough photons under various settings of  $A_1$  and  $A_2$ , we can determine  $\Delta(A_1, A_2)$ , in principle to arbitrary accuracy. This permits an experimental check on which assumption gives an adequate description of the situation.

**Analysis with hidden variables.** For the analysis with classical hidden variables, we make the following assumptions:

(i) The source of beam 1 produces an ensemble of photons which is in the classical (but submicroscopic) state  $\lambda$  with probability density  $p(\lambda)$ .

(ii) Whether a photon created at the source in state  $\lambda$  reaches the detector after passing the  $k$ th filter depends only on  $A_k$  and  $\lambda$ . (This is reasonable since, initially, beam 2 is completely dark and hence carries no photons.)

(iii) The conditional probability of detecting a photon which is in state  $\lambda$  and passes through filter  $k$  when  $A_k = A$  and  $A_{3-k} = 0$  is  $p_k(A, \lambda)$ .

$p(\lambda)$  and  $p_k(A, \lambda)$  are determined by the whole, fixed arrangement. Under these assumptions, the probability of detection of a photon when  $A_1$  and  $A_2$  are arbitrary is

$$\begin{aligned} p(A_1, A_2) &= \int d\lambda p(\lambda) (p_1(A_1, \lambda) + p_2(A_2, \lambda)) \\ &= \int d\lambda p(\lambda) p_1(A_1, \lambda) + \int d\lambda p(\lambda) p_2(A_2, \lambda) \\ &= p(A_1, 0) + p(0, A_2), \end{aligned}$$

hence we get the

$$\text{hidden variable prediction:} \quad \Delta(A_1, A_2) = 0. \quad (2)$$

**Analysis by quantum mechanics.** Assuming quantum mechanics, we have, with the notation from the figure,

$$\begin{aligned}\psi'_1 &= t_1^a \psi_1 + r_2^a \psi_2, & \psi'_2 &= r_1^a \psi_1 + t_2^a \psi_2, \\ \psi''_1 &= A_1 \psi'_1, & \psi''_2 &= A_2 \psi'_2, \\ \psi_D &= t_1^b \psi''_1 + r_2^b \psi''_2.\end{aligned}$$

Since the second beam is initially dark,  $\psi_2 = 0$ , and we find

$$\psi_D = t_1^a t_1^b A_1 \psi_1 + r_1^a r_2^b A_2 \psi_1.$$

From this, we find

$$p(A_1, A_2) = q |\psi_D|^2 = q |t_1^a t_1^b A_1 \psi_1 + r_1^a r_2^b A_2 \psi_1|^2.$$

Evaluating (1) and simplifying, we end up with the

$$\text{quantum prediction:} \quad \Delta(A_1, A_2) = 2q |t_1^a t_1^b r_1^a r_2^b|^2 \text{Re} \psi_1^* A_1^* A_2 \psi_1. \quad (3)$$

### 3 Discussion

Upon comparing (2) and (3), we see that the prediction (3) of quantum mechanics differ significantly from the prediction (2) of any hidden variable theory satisfying our assumptions. The nonlinearity in the squared amplitude formula for the probability is responsible for a nontrivial interference term. Thus, comparable to destructive interference in two-slit experiments, constructive interference is the source for the discrepancy between (3) and (2). (This is an instance of a more general phenomenon discussed by MALLEY [18] in a more abstract context, that – under much stronger assumptions – hidden variables imply the absence of quantum interference terms.)

In terms of the density matrix  $\rho = \psi_1 \psi_1^*$ , the quantum prediction can be expressed as

$$\Delta(A_1, A_2) = 2q |t_1^a t_1^b r_1^a r_2^b|^2 \text{Re tr}(\rho A_1^* A_2).$$

This relation remains valid if, in place of a pure state  $\psi_1$ , the source produces photons prepared in an arbitrary mixed state  $\rho$ , normalized such that  $\text{tr} \rho$  equals the mean number of photons in the fixed time interval  $[t_0, t_1]$ .

Since the experiment does not involve photon correlation measurements, the quantum analysis even holds for multiphoton input, provided one takes  $\rho$  as the effective single-photon density matrix of the multiphoton state, again normalized such that  $\text{tr} \rho$  equals the mean number of photons in the fixed time interval  $[t_0, t_1]$ .

This allows the experiment to be carried out with strong laser light. In this case, the number of photons is enormous, and the probabilities turn into essentially deterministic current strengths. Thus performing the experiment will leave no doubt about the decision for or against hidden variables, in contrast to the presence of detection loopholes in current experiments on local hidden variable theories; cf., e.g., GENOVESE [9] or BABICHEV et al. [2].

The actual performance of the experiment is expected to reproduce the quantum predictions, thus excluding a theory satisfying our hidden variable assumptions.

Note, however, that the experiment can be explained by the classical Maxwell equations (as discussed in the book by MANDEL & WOLF [19]), upon interpreting the photon number detection rate as proportional to the beam intensity. This is a classical description, not by classical particles (photons) but by classical waves.

Indeed, it is well-known (see, e.g., WEINBERG [29]) that Maxwell's theory in vacuum can be regarded as the theory of a classical zero mass spin 1 photon field, whose quantization (together with that of a classical spin 1/2 electron field) gives quantum electrodynamics (QED). Moreover (cf. MANDEL & WOLF [19, Chapter 9]), the photoeffect can be explained already with classical light (and quantized electrons), hence is independent of any quantum notion of photons.

In this light, the present analysis demonstrates that a classical wave model for quantum mechanics is not ruled out by experiments demonstrating the violation of the traditional hidden variable assumptions.

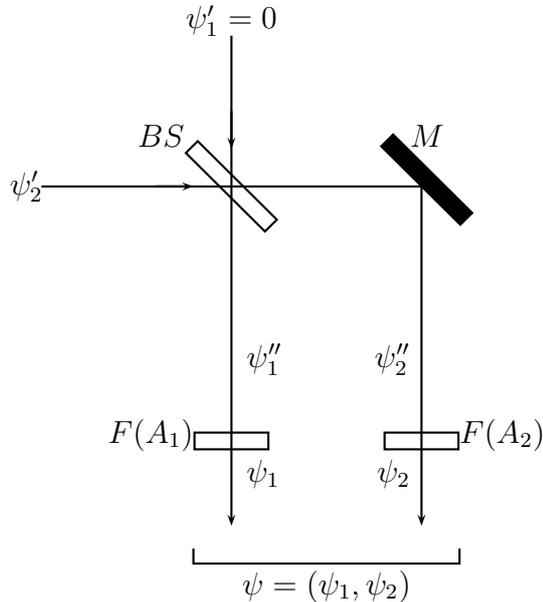
In particular, this diminishes the role Bell inequality violations play for investigations of the foundations of quantum physics. From the new perspective gained by the present analysis, the traditional hidden variable assumptions therefore only amount to *hidden particle assumptions*, and the experiments demonstrating their violation are just another chapter in the old dispute between the particle or field nature of light (cf. MUTHUKRISHNAN et al. [20, p. 20]), conclusively resolved in favor of the field.

## 4 General entangled states

It is fairly easy to see that general entangled states can be prepared and measured by the arrangements in Figures 2 and 3 obtained by splitting our experiment into two halves; cf. KIM [16] for alternative preparation and measurement settings.

With more beam splitters, through which several narrowly spaced beams are passed, one can produce a cascade of more complex tensor product states. Indeed, RECK et al. [23] showed that (i) any quantum system with only finitely many degrees of freedom can be

Figure 2: State preparation. The first input beam is assumed to be dark.



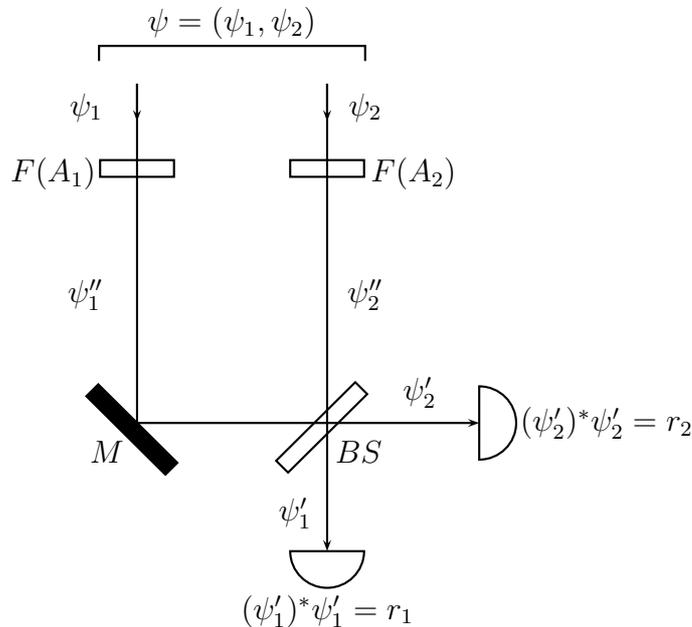
simulated by a collection of spatially entangled beams; (ii) in the simulated system, there is for any Hermitian operator  $H$  an experiment measuring  $H$ ; (iii) for every unitary operator  $S$ , there is an optical arrangement in the simulated system realizing this transformation, assuming lossless beam splitters.

It is not very difficult to show along the lines of [23] that, with additional polarizers and with our convention of scaling state vectors to reflect the probability of presence of a photon, one can similarly realize every subunitary operator  $S$ , characterized by the condition that all eigenvalues of  $S^*S$  are bounded by 1.

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Figure 3: State detection. The input state is unknown. Output entanglement is not measured. Input entanglement can be inferred by measuring with different settings of  $A_1$  and  $A_2$ .



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